Color suppressed contributions to the decay modes $B_{d,s} \to D_{s,d} D_{s,d}$, $B_{d,s} \to D_{s,d} D^*_{s,d}$, and $B_{d,s} \to D^*_{s,d} D^*_{s,d}$.

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ABSTRACT

The amplitudes for decays of the type $B_{d,s} \to D_{s,d} D_{s,d}$, have no factorizable contributions, while $B_{d,s} \to D_{s,d} D^*_{s,d}$, and $B_{d,s} \to D^*_{s,d} D^*_{s,d}$ have relatively small factorizable contributions through the annihilation mechanism. The dominant contributions to the decay amplitudes arise from chiral loop contributions and tree level amplitudes which can be obtained in terms of soft gluon emissions forming a gluon condensate. We predict that the branching ratios for the processes $\bar{B}^0_d \to D^+_s D^-_s$, $\bar{B}^0_d \to D^{+*}_s D^-_s$ and $\bar{B}^0_d \to D^+_s D^{-*}_s$ are all of order $(2-3) \times 10^{-4}$, while $\bar{B}^0_s \to D^+_d D^-_d$, $\bar{B}^0_s \to D^{+*}_d D^-_d$ and $\bar{B}^0_s \to D^+_d D^{-*}_d$ are of order $(4-7) \times 10^{-3}$. We obtain branching ratios for two D^*'s in the final state of order two times bigger.

1 Introduction

Recently, many theoretical and experimental studies in B-meson physics have been done. The most extensive investigations have been done for cases of B-decay modes into the light mesons (e.g. $B \to \pi\pi$, $B \to K\pi$) due to the their importance in the determination of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. On the other hand B-decays into charmed mesons also present an important issue for experimental and theoretical analysis. As it was pointed out by the authors of [1, 2], final state interactions (FSI) including charmed intermediate states can give significant contributions to the decay amplitudes, especially in the case of the $B \to K\pi$ decay modes [1]. In addition, branchig ratios for B_d decay into $D^- D_s^+$, $D^{-*} D_s^+$, $D^- D_s^{*+}$ and $D^{*-} D_s^{*+}$ [3] have been measured. A previous theoretical study of the decay mode $B_d \to D^- D_s^+$ [4] showed that the non-factorizable contributions coming from chiral loops give an increase of order 10% in the rate compared to the factorized limit. The rate found in [4] is in good agreement with the experimental result. On the other hand, the $\bar{B}^0 \to D_s^+ D_s^-$ and $\bar{B}_s^0 \to D^+ D^-$ decay modes have no factorizable amplitudes and they are realized only through non-factorizable contributions as it was shown in [5]. At the quark level, these decays a priori proceed through the annihilation mechanism $b\bar{s} \to c\bar{c}$ and $b\bar{d} \to c\bar{c}$, respectively. Within the factorized limit this mechanism will give a zero amplitude due to current conservation, as in the case of $D^0 \to K^0 \overline{K^0}$ [6]. Therefore $B_{d,s} \to D_{s,d} D_{s,d}$, $B_{d,s} \to D_{s,d} D^*_{s,d}$, and $B_{d,s} \to D^*_{s,d} D^*_{s,d}$ present a fertile ground for the investigation of non-factorizable $1/N_c$ suppressed contributions.

In this paper we want to extend our previous study of $B_{d,s} \to D_{s,d} D_{s,d}$ decays to cases with one or two D^*'s in the final state. Namely, at B-factories the decays to one pseudoscalar and one vector D-meson are easier accessable due to better statistical accuracy (they can be reconstructed more inclusively than decays to two pseudoscalars) [7]. In this case non-factorizable contributions arise due to the annihilation mechanism. However, its amplitude is suppressed by a numerically less favorable combination of Wilson coefficients and is expected to be of the same order of magnitude as non-factorizable contributions. Since the energy release for B-decays to two charm mesons is relatively small (of order 1 GeV), the QCD improved approaches [8] used to describe B-decays to light mesons are not expected to hold. Therefore, we develop a different approach.

As in [5] our framework will be threefold: We use the standard effective Lagrangian approach for the quark process $b\overline{q} \to c\overline{c}$ (where $q = d, s$) found at a scale below m_c [9]. Second, we use heavy-light chiral perturbation theory for interactions between heavy mesons and light pseudoscalar mesons [10] to calculate non-factorizable contributions in terms of chiral loops. Third, to estimate the contributions from $1/N_c$ suppressed terms at tree level [11, 12] within heavy-light chiral perturbation theory, we use a recently developed Heavy Light Chiral Quark Model (HLχQM) [13] based on the Heavy Quark Effective Field Theory (HQEFT) [14].

Within our approach, the chiral symmetry breaking scale $\Lambda_\chi \sim 1$ GeV is the matching scale for pertubative QCD, chiral perturbation theory, and HLχQM. The latter is also the bridge between them. In the next section 2, we describe our framework. In section 3 we present the factorizable (annihilation), gluon condensate and chiral loop contributions to the amplitudes. In section 4 we give the numerical results and conclusions.

2 Framework

2.1 Effective Lagrangian at quark level

Based on the electroweak and quantum chromodynamical interactions, one constructs an effective Lagrangian at the quark level in a standard well known way:

$$\mathcal{L}_W = \sum_i C_i(\mu)\, Q_i(\mu)\,, \tag{1}$$

where $C_i(\mu) = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* \, a_i(\mu)$, $q = d, s$ and $a_i(\mu)$ are dimensionless Wilson coefficients that carry all information of the short distance (SD) physics above the renormalization scale μ. The matrix elements of the operators $Q_i(\mu)$ on the other hand, take care of all non-perturbative, long distance (LD) physics below μ. The relevant operators in our case are

$$Q_1 = 4(\overline{q}_L \gamma^\alpha b_L)\;(\overline{c}_L \gamma_\alpha c_L)\,, \qquad Q_2 = 4(\overline{c}_L \gamma^\alpha b_L)\;(\overline{q}_L \gamma_\alpha c_L)\,, \tag{2}$$

where L denotes a left-handed particle. Contributions from other (say, penguin) operators are neglected due to smallness of their Wilson coefficients.

In order to obtain all matrix elements of the Lagrangian (1) we need the Fierz transformed version of the operators in (2). To find these, we use the relation:

$$\delta_{ij}\delta_{ln} = \frac{1}{N_c}\delta_{in}\delta_{lj} \;+\; 2\; t^a_{in}\; t^a_{lj}\,, \tag{3}$$

where i, j, k and n are color indices running from 1 to 3 and a is a color octet index. One obtains

$$Q_1^F = \frac{1}{N_C} Q_2 + 2\widetilde{Q}_2\,, \quad Q_2^F = \frac{1}{N_C} Q_1 + 2\widetilde{Q}_1\,, \tag{4}$$

where the superscript F stands for "Fierzed", and

$$\widetilde{Q_1} = 4(\overline{q}_L \gamma^\alpha t^a b_L)\;\;(\overline{c}_L \gamma_\alpha t^a c_L)\,, \qquad \widetilde{Q_2} = 4\,(\overline{c}_L \gamma^\alpha t^a b_L)\;\;(\overline{q}_L \gamma_\alpha t^a c_L)\,, \tag{5}$$

where t^a denotes the color matrices.

In order to calculate the matrix elements of the operators $\widetilde{Q}_1$ and $\widetilde{Q}_2$ we will use a version of the Heavy Light Chiral Quark Model (HLχQM) developed in [13]. It belongs to a class of models extensively studied in the literature [15] - [21] and is appropriate for describing interactions *in which the transferred enegry is* of the order 1 GeV. This sets the scale in (1) to $\mu \sim \Lambda_\chi \sim 1\,\text{GeV}$, which is by construction the matching scale within our approach. At this scale one finds $a_1 \simeq -0.35 - 0.07i$ and $a_2 \simeq 1.29 + 0.08i$ [4, 9]. Note that the a_i's are complex below the charm scale.

2.2 Heavy light chiral perturbation theory

The construction of heavy light chiral perturbation theory is based on the heavy quark effective theory (HQEFT) [14], which is a systematic $1/m_Q$ expansion in the heavy quark mass m_Q. The

Lagrangian is obtained by replacing the heavy quark Dirac field $Q(x) = b(x), c(x)$ or $\overline{c}$ with a "reduced" field $Q_v^{(+)}(x)$ for a heavy quark, and $Q_v^{(-)}(x)$ for a heavy anti-quark. These are related to the full field $Q(x)$ in the following way:

$$Q_v^{(\pm)}(x) = P_\pm e^{\mp i m_Q v\cdot x} Q(x) \,, \tag{6}$$

where $P_\pm = (1 \pm \gamma \cdot v)/2$ are projecting operators and v is the velocity of the heavy quark. The Lagrangian for heavy quarks then reads:

$$\mathcal{L}_{HQEFT} = \pm \overline{Q_v^{(\pm)}} \, iv \cdot D \, Q_v^{(\pm)} + \mathcal{O}(m_Q^{-1}) \,, \tag{7}$$

where D_μ is the covariant derivative containing the gluon field and $\mathcal{O}(m_Q^{-1})$ stands for the $1/m_Q$ corrections [12], which will not be considered in this paper.

After integrating out the heavy and light quarks, the effective Lagrangian for heavy and light mesons up to $\mathcal{O}(m_Q^{-1})$ can be written as [13, 22]:

$$\mathcal{L} = \mp Tr \left[\overline{H_a^{(\pm)}} iv \cdot \mathcal{D}_{ba} H_b^{(\pm)} \right] - g_{\mathcal{A}} \, Tr \left[\overline{H_a^{(\pm)}} H_b^{(\pm)} \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba} \right] + ... \,, \tag{8}$$

where $H_a^{(\pm)}$ is the heavy meson field containing a spin zero and a spin one boson:

$$H_a^{(\pm)} \quad \equiv P_\pm (P_{a\mu}^{(\pm)} \gamma^\mu - i P_{a5}^{(\pm)} \gamma_5) \ . \tag{9}$$

The field $P_M^{(+)} (P_M^{(-)})$ ($M = \mu$ for a vector and $M = 5$ for a pseudo-scalar) annihilates (creates) a heavy meson containing a heavy quark (anti-quark) with velocity v. Furthermore, $\mathcal{D}^\mu_{ba} = \delta_{ba} D^\mu - \mathcal{V}^\mu_{ba}$, where a, b are flavor indices. The vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are defined as:

$$\mathcal{V}_\mu \equiv \frac{i}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \,, \quad \mathcal{A}_\mu \equiv -\frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \,, \quad \xi \equiv exp[i\Pi/f] \,, \tag{10}$$

where f is the bare pion coupling, and Π is a 3 by 3 matrix which contains the Goldstone bosons π, K, η in the standard way. The ellipses in (8) denote terms of higher order in the chiral expansion.

Based on the symmetry of HQEFT, we can obtain the bosonized currents. For a decay of the $b\bar{q}$ system we have [13, 22]:

$$\overline{q_L} \, \gamma^\mu \, Q_{v_b}^{(+)} \ \longrightarrow \ \frac{\alpha_H}{2} Tr \left[\xi^\dagger \gamma^\alpha L \, H_b^{(+)} \right] \,, \tag{11}$$

where $\alpha_H = f_H \sqrt{m_H}$. In the limit $m_Q \to \infty$, $\alpha_B = \alpha_D = \alpha_H$, but there are $1/m_Q$ and perturbative QCD corrections to this limit [13, 14]. Here $Q_{v_b}^{(+)}$ is the heavy b-quark field, v_b is its velocity, and $H_b^{(+)}$ is the corresponding heavy meson field.

For the W-boson materializing to a $\bar{D}$-meson we obtain:

$$\overline{q_L} \gamma^\mu \, Q_{\bar{v}}^{(-)} \ \longrightarrow \ \frac{\alpha_H}{2} Tr \left[\xi^\dagger \gamma^\alpha L H_{\bar{c}}^{(-)} \right] \,, \tag{12}$$

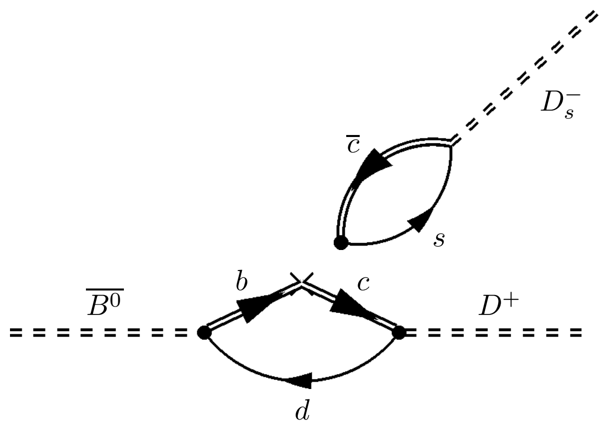

Figure 1: Factorized contribution for $\overline{B^0} \to D^+ D_s^-$ through the spectator mechanism, which does not exist for decay mode $\overline{B^0} \to D_s^+ D_s^-$.

where $Q_{\bar{v}}^{(-)}$ and $\bar{v}$ $(= v_{\bar{c}})$ is the heavy quark field and the velocity of the $\bar{c}$ quark, respectively, and $H_{\bar{c}}^{(-)}$ is the corresponding field for the $\bar{D}$-meson. Similar, for the $b \to c$ transition, the bosonized current is:

$$\overline{Q_{v_b}^{(+)}} \gamma^\mu L Q_{v_c}^{(+)} \longrightarrow -\zeta(\omega) Tr \left[\overline{H_c^{(+)}} \gamma^\alpha L H_b^{(+)} \right] , \tag{13}$$

where $Q_{v_c}^{(+)}$ and v_c is the heavy quark field and the velocity of the c-quark, respectively. Furthermore, $\zeta(\omega)$ is the Isgur-Wise function for the $\bar{B} \to D$ transition, $H_c^{(+)}$ the heavy D-meson field(s), and $\omega \equiv v_b \cdot v_c = v_b \cdot \bar{v} = 1/(2\kappa)$, where $\kappa \equiv M_D/M_B$.

For the weak current for $D\bar{D}$ production (corresponding to the factorizable annihilation mechanism) we obtain:

$$\overline{Q_{v_c}^{(+)}} \gamma^\mu L Q_{\bar{v}}^{(-)} \longrightarrow -\zeta(-\lambda) Tr \left[\overline{H_c^{(+)}} \gamma^\alpha L H_{\bar{c}}^{(-)} \right] , \tag{14}$$

where $\lambda = \bar{v} \cdot v_c = (1/(2\kappa^2) - 1)$, and $\zeta(-\lambda)$ is a (complex) function less studied and not so well known as $\zeta(\omega)$ in (13).

The factorized contributions for the spectator and annihilation diagrams are shown in Figs. 1 and 2. The first diagram does not give any (direct) contributions to the class of processes we consider, but is still important because it is the basis for our chiral loops.

The non-factorizable chiral loop contributions to the amplitudes can be visualized in Fig. 3, where the heavy pseudo-scalar mesons are represented with single lines, the heavy vector mesons with double lines, the light kaons with dashed lines, and the weak vertex with two circles. As seen from Fig. 3, the chiral loop contribution can be divided into two topologies

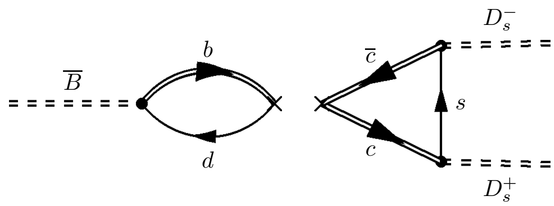

Figure 2: Factorized contribution for $\overline{B^0} \to D_s^+ D_s^-$ through the annihilation mechanism, which give zero contributions if both D_s^+ and D_s^- are pseudoscalars.

that we will denote as topology I and topology II, respectively. In the topology I (diagrams A_{01}, A_{11}, A_{21}, A_{22}, $A_{31}(A_{41})$, and $A_{32}(A_{42})$ in Fig. 3) the B-meson radiates $\bar{K}^0$ becoming a $b\bar{s}$ state that decays into $D^-(D^{-*})$ and $D_s^+(D_s^{+*})$. The $\bar{K}^0$ meson is then reabsorbed by $D^-(D^{-*})$ giving the $D_s^-(D_s^{-*})$ state. In the topology II (diagrams A_{02}, A_{12}, A_{13}, A_{23}, A_{24}, $A_{33}(A_{43})$, $A_{34}(A_{44})$, and $A_{35}(A_{45})$ in Fig. 3), the B-meson decays into $D^-(D^{-*})$ and $D^+(D^{+*})$ which then re-scatter by means of a kaon into $D_s^-(D_s^{-*})$ and $D_s^+(D_s^{+*})$. In both cases, the amplitudes for intermediate $\bar{B}^0 \to D^+D^-$ or $\bar{B}_s \to D^-D_s^+$ processes (and similar for one or two vectors in a final state) can, within the factorized limit, be written as the product of currents (12) and (13) multiplied by the numerically favorable Wilson coefficient combination $(C_2 + C_1/N_c)$. Vertices describing absorption or radiation of a kaon are given by (8). There are of course also factorizable loop contributions, but these are included in the decay constants $f_{D,B}$ and the Isgur-Wise functions.

The calculation of the chiral loop amplitudes includes the calculation of a divergent integral of a form:

$$I(v_1, v_2)^{\sigma\rho} \;=\; \frac{1}{4}\int \frac{d^D k}{(2\pi)^D} \frac{k^\sigma\, k^\rho}{(k\cdot v_1 + i\epsilon)(k\cdot v_2 + i\epsilon)(k^2 - m_K^2 + i\epsilon)}\,. \tag{15}$$

In the dimensional regularization method, the integral can be rewritten as:

$$I(v_1, v_2)^{\sigma\rho} \;=\; \frac{1}{4} I_1 \left[-r\, g_D^{\sigma\rho} + \frac{r - x}{1 - x^2}(v_1^\sigma v_1^\rho + v_2^\sigma v_2^\rho) + \frac{1 - x\, r}{1 - x^2}(v_1^\sigma v_2^\rho + v_2^\sigma v_1^\rho) \right], \tag{16}$$

where $g_D^{\sigma\rho}$ is a D dimensional matrix tensor and

$$I_1 \equiv \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 - m_K^2} = \frac{i m_K^2}{16\pi^2}\left[\Delta - \ln\frac{m_K^2}{\mu^2} + 1\right], \tag{17}$$

with $\Delta = 2/(4-D) - \gamma_E + \ln 4\pi$. Here $r = r(x)$ with $x = v_1 \cdot v_2$ is a function defined by:

$$r(x) = \frac{1}{\sqrt{x^2 - 1}} \ln\left(|x| + \sqrt{x^2 - 1}\right), \tag{18}$$

for $x > 0$, and

$$r(x) = -\frac{1}{\sqrt{x^2 - 1}}(\ln\left(|x| + \sqrt{x^2 - 1}\right) - i\pi), \tag{19}$$

for $x < 0$.

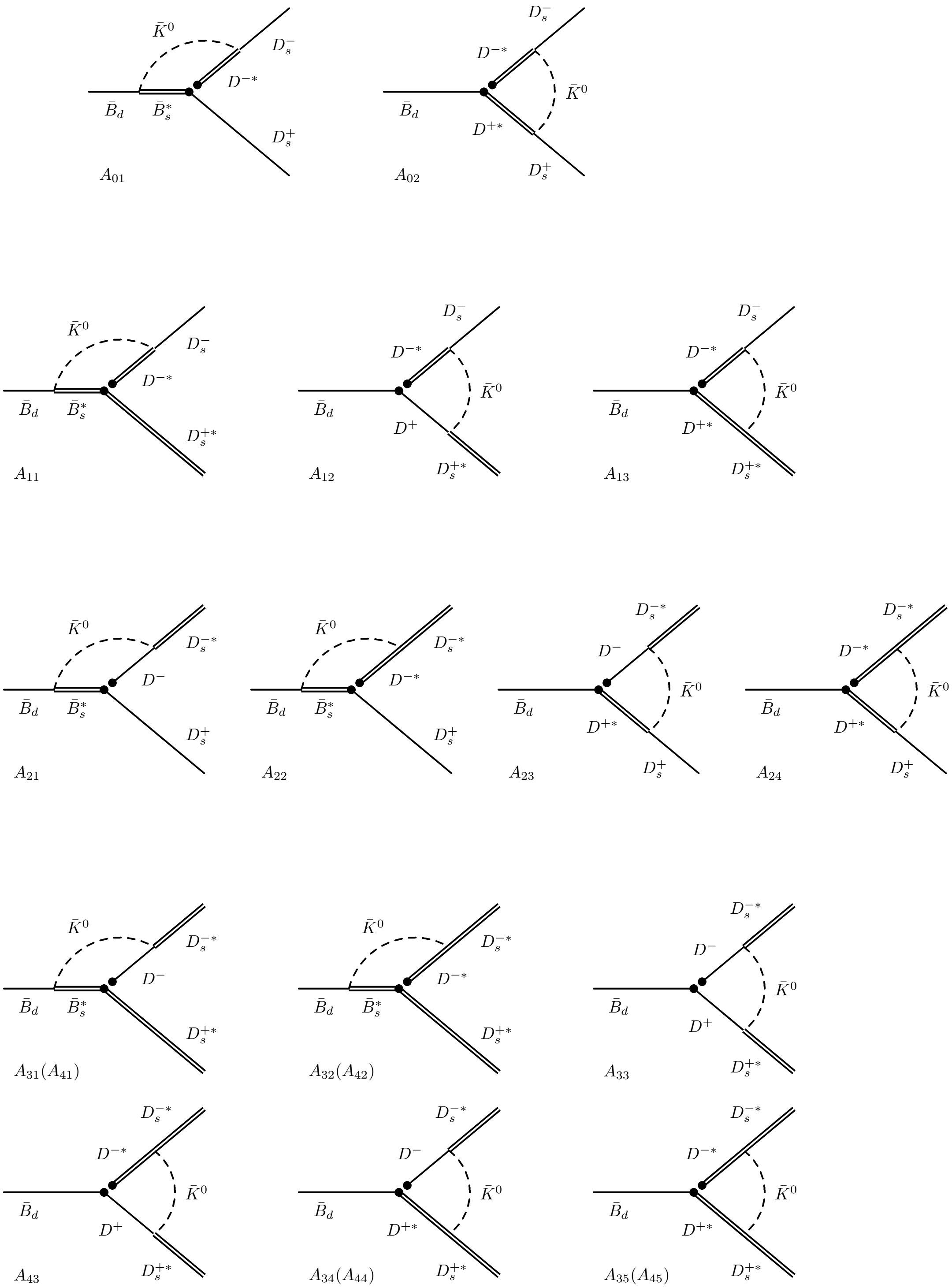

Figure 3: Feynman diagrams for the chiral loop contributions. For $\overline{B^0} \to D_s^{+*} D_s^{-*}$, the diagrams A_{4i} stand for contributions proportional to the Levi-Civita term.

2.3 The heavy light chiral quark model (HLχQM)

The HLχQM Lagrangian can be written as

$$\mathcal{L}_{\mathrm{HL}\chi\mathrm{QM}} = \mathcal{L}_{HQEFT} + \mathcal{L}_{\chi QM} + \mathcal{L}_{Int} \; . \tag{20}$$

The first term describes the interaction of heavy quarks in (7). The second term describes the interactions of light quarks with light (Goldstone) mesons in terms of the chiral quark model (χQM) [15, 16, 24, 25, 26]:

$$\mathcal{L}_{\chi QM} = \bar{\chi} \left[\gamma^\mu (i D_\mu + \mathcal{V}_\mu + \gamma_5 \mathcal{A}_\mu) - m \right] \chi \; . \tag{21}$$

Here $m = 0.23 \pm 0.02\,\mathrm{GeV}$ is the $SU(3)$ invariant constituent light quark mass, and χ is the flavor rotated quark fields given by $\chi_L = \xi^\dagger q_L$ and $\chi_R = \xi q_R$, where $q^T = (u, d, s)$ are the light quark fields. The left- and right-handed projections q_L and q_R are transforming after $SU(3)_L$ and $SU(3)_R$ respectively. In (21) we have discarded terms involving the light current quark mass which are irrelevant in the present paper (but become important when calculating counterterms). The covariant derivative D_μ in (21) contains the soft gluon field forming the gluon condensates. The effects based on (21) can be calculated by Feynman diagram techniques as in [11, 12, 13, 24, 25] or by the means of heat kernel techniques as in [16, 21, 26].

The interaction between heavy meson fields and quarks is described by [13, 17, 18, 20, 19]:

$$\mathcal{L}_{Int} = -G_H \left[\overline{\chi}_a \, \overline{H_a^{(\pm)}} \, Q_v^{(\pm)} + \overline{Q_v^{(\pm)}} \, H_a^{(\pm)} \, \chi_a \right] \; , \tag{22}$$

with the coupling constant $G_H = \sqrt{2m\rho}/f$, where ρ is a hadronic parameter (of the order one) depending on m [13].

Within the model, one finds the following expression for the Isgur-Wise function [13]:

$$\zeta(\omega) = \frac{2}{1+\omega} \, (1-\rho) + \rho \, r(\omega) \, , \tag{23}$$

where $r(\omega)$ is given by (18). In the simple expression (23) chiral loop and perturbative QCD corrections down to the scale $\mu = \Lambda_\chi$, have not been taken into account. Nevertheless, it gives a good description of the Isgur-Wise function [13]. For negative values of the argument the Isgur-Wise function $\zeta(-\lambda)$ is, within the HLχQM [13], still given by (23) with $\omega \to -\lambda$.

Performing the bosonization of the HLχQM, one encounters divergent loop integrals which will in general be quadraticly, linearly and logarithmicly divergent [13]. The quadraticly and logarithmicly divergent integrals are related to the quark condensate and the bare pion decay constant f [25], respectively. The linearly divergent integral (which is finite within dimensional regularization) is related to the axial coupling $g_{\mathcal{A}}$ in (8).

The gluon condensate contribution to the amplitudes can be written, within the framework presented in the previous section, in a quasi-factorized way as a product of matrix elements of colored currents:

$$\langle D_s^+ D_s^- | \mathcal{L}_W | \overline{B^0} \rangle_{NF}^G \; = \; 8 C_2 \, \langle D_s^+ D_s^- | \overline{c}_L \gamma_\mu t^a c_L | G \rangle \langle G | \overline{d}_L \gamma^\mu t^a b_L | \overline{B^0} \rangle \; , \tag{24}$$

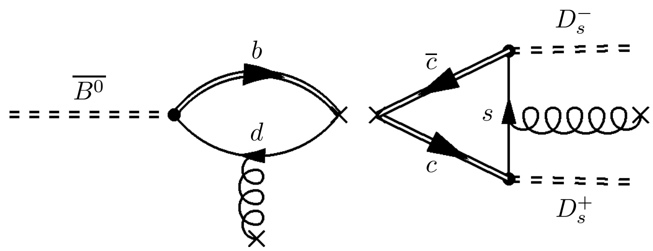

Figure 4: Non-factorizable contribution for $\overline{B^0} \to D_s^+ D_s^-$ through the annihilation mechanism with additional soft gluon emision. The wavy lines represent soft gluons ending in vacuum to make gluon condensates.

where G in the bra-kets symbolizes emission of a gluon as visualized in Fig. 4. The left part in Fig. 4 gives us the bosonized colored current:

$$\left(\overline{q_L}\, t^a\, \gamma^\alpha\, Q_{v_b}^{(+)}\right)_{1G} \;\longrightarrow\; -\frac{G_H\, g_s}{64\pi}\, G_{\mu\nu}^a Tr\left[\xi^\dagger \gamma^\alpha L\, H_b^{(+)}\, (\sigma^{\mu\nu} \;-\; F\,\{\sigma^{\mu\nu}, \gamma\cdot v_b\}\,)\right] , \tag{25}$$

where $G_{\mu\nu}^a$ is the octet gluon tensor, and $F \;\equiv\; 2\pi f^2/(m^2\, N_c)$ is a dimensionless quantity of the order 1/3. The symbol $\{\ ,\ \}$ denotes the anti-commutator.

For the creation of a $D\bar{D}$ pair in the right part of Fig. 4 (the analogue of (25)) one gets:

$$\begin{aligned}\left(\overline{Q_{v_c}^{(+)}}\, t^a\, \gamma^\alpha\, L Q_{\bar{v}}^{(-)}\right)_{1G} \;\longrightarrow\; & \frac{G_H^2\, g_s}{128\pi m(\lambda-1)}\, G_{\mu\nu}^a\; Tr\left[\overline{H_c^{(+)}}\, \gamma^\alpha\, L\, H_{\bar{c}}^{(-)}\right.\\ & \left.\times\, (X \sigma^{\mu\nu} \;+\; \{\sigma^{\mu\nu}, \gamma\cdot\Delta v\}\,)\right] ,\end{aligned} \tag{26}$$

where

$$X \equiv \frac{4}{\pi}(\lambda-1)\, r(-\lambda) \, , \tag{27}$$

and $\Delta v = v_c - \bar{v}$. Multiplying the currents in Eqs. (25) and (26), and using the replacement:

$$g_s^2 G_{\mu\nu}^a G_{\alpha\beta}^a \;\to 4\pi^2 \langle\frac{\alpha_s}{\pi} G^2\rangle \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \, , \tag{28}$$

we obtain a bosonized effective Lagrangian term which is $1/N_c$ suppressed compared to the factorized contributions. This effective Lagrangian term corresponds to a certain linear combination of a priori possible $1/N_c$ suppressed terms at the tree level in the chiral perturbation theory sense.

3 Calculating the decay amplitudes

First, for comparison, we give the factorized amplitude for the process $\overline{B^0} \to D^+ D_s^-$

$$\mathcal{A}(\overline{B^0} \to D^+ D_s^-)_F = (C_2 + \frac{C_1}{N_c})\, \zeta(\omega) f_D M_D \sqrt{M_B M_D}\; (\lambda + \omega) \ . \tag{29}$$

The $\bar{B}^0 \to M\bar{M}$, $M = D_s^+, D_s^{+*}$ decay amplitudes can be written in the following form:

$$\mathcal{A}(B \to D\bar{D}) = F_0^{PP}, \qquad \mathcal{A}(B \to D^*\bar{D}) = iF_0^{VP}\ \varepsilon_{D^*} \cdot \bar{v}\,, \tag{30}$$

$$\mathcal{A}(B \to D\bar{D}^*) = iF_0^{VP}\ \varepsilon_{\bar{D}^*} \cdot v_c\,, \tag{31}$$

$$\mathcal{A}(B \to D^*\bar{D}^*) = F_0^{VV}\ \varepsilon_{D^*} \cdot \varepsilon_{\bar{D}^*} + F_1^{VV}\ \varepsilon_{D^*} \cdot \bar{v}\ \varepsilon_{\bar{D}^*} \cdot v_c + F_2^{VV}\ i\epsilon_{\alpha\beta\mu\nu} v_c^\alpha \bar{v}^\beta \varepsilon_{D^*}^\mu \varepsilon_{\bar{D}^*}^\nu\,, \tag{32}$$

with the reduced amplitudes F_i^{NN}, containing the contributions coming from the tree level factorizable contributions $F_{i,fc}^{NN}$, gluon condensates $F_{i,gc}^{NN}$ and chiral loops $F_{i,cl}^{NN}$:

$$F_i^{NN} = F_{i,fc}^{NN} + F_{i,gc}^{NN} + F_{i,cl}^{NN}\ , \tag{33}$$

where $N = P, V$ mean pseudoscalar D and vector meson D^*, respectively.

The factorizable amplitude for $\overline{B^0} \to M\bar{M}$, $M = D_s^+, D_s^{+*}$ comes from the annihilation diagram:

$$\langle D_s^- D_s^+ | \mathcal{L}_W | \overline{B^0} \rangle_F \ = \ 4(C_1 + \frac{1}{N_c} C_2) \langle D_s^- D_s^+ | \overline{c_L} \gamma_\mu c_L | 0 \rangle \langle 0 | \overline{d_L} \gamma^\mu b_L | \overline{B^0} \rangle\ , \tag{34}$$

and is proportional to a numerically non-favorable combination of Wilson coefficients. Using Eqs. (11) and (14) we obtain the following values for the reduced amplitudes:

$$F_{0,fc}^{PV} = -F_{0,fc}^{VP} = F_{2,fc}^{VV} \sqrt{\frac{M_D}{M_{D^*}}} \ = \ 2\,(C_1 + \frac{C_2}{N_c})\,\zeta(-\lambda) \alpha_B\, \kappa\ \sqrt{M_B M_D M_{D^*}}\,, \tag{35}$$

$$F_{0,fc}^{PP} = F_{0,fc}^{VV} = F_{1,fc}^{VV} = 0\,. \tag{36}$$

Using (25) - (28), we obtain the gluon condensate contributions (the color suppressed $1/N_c$ contributions at the tree level) to the reduced amplitudes illustrated in Fig. 4:

$$F_{0,gc}^{PP} \ = \ 3\,S\,\left(X + \frac{4}{3}(\lambda - 1) \right)\,, \qquad F_{0,gc}^{VP} \ = \ S\ (X\,[1 - 2F] + 4\,[\lambda + 2F])\ , \tag{37}$$

$$F_{0,gc}^{PV} \ = \ S\ (X\,[3 + 2F] + 4\,[(\lambda - 2) - 2F])\ , \qquad F_{0,gc}^{VV} \ = \ 2\kappa^2 S\, X\,(\lambda + 1)\,, \tag{38}$$

$$F_{1,gc}^{VV} \ = \ -2\kappa^2 S\,(X - 4)\,, \qquad F_{2,gc}^{VV} \ = \ 2\kappa^2 S\,(X - 4)(1 + 2F)\,, \tag{39}$$

where F is defined below (25), X is defined in (27), and

$$S \ \equiv \ \frac{C_2\,\left(G_H \sqrt{M_B} \right)^3}{3 \cdot 2^9 m(\lambda - 1)}\ \langle \frac{\alpha_s}{\pi} G^2 \rangle\,. \tag{40}$$

In the evaluation of the integrals (16) and (17) appearing in the reduced chiral loops amplitudes $F_{i,cl}^{NN}$, we use the $\overline{MS}$ scheme and we take $\mu \simeq \Lambda_\chi \simeq 1$ GeV. The integral (17) contains a logarithmicly divergent term and a constant term. However, additional contributions to the

constant term might come from counterterms. The finite part of these terms is unknown and therefore, in our numerical computation, we take into account logarithmic terms only, which are independent of the counterterms contributions, and we consider constant term contributions as theoretical uncertainities of our approach.

However, the products of two Levi-Civita terms enter in our computation of the amplitudes coming from diagrams $A_{13}\,, A_{22}\,, A_{32}\,, A_{35}$ and A_{45}. Whitin dimensional regularization, these products are not uniquelly defined. This is related to the problem known in the literature as γ_5 scheme dependance [27]. However, this scheme dependence appears in the constant terms only. To estimate its influence on numerical results, we use two different appraches to the products of two Levi-Civita symbols. (One is the dimensoinal reduction [27] and the second one is a variation of the dimensional reduction in which the products of metric tensors in D dimension and the metric tensor in 4-dimensions are fixed on the 4-dimensional space.)

The logarithmic contributions to the reduced amplitudes are:

$$
\begin{aligned}
F_{0,cl}^{PP} &= iK\sqrt{\frac{M_D}{M_D^*}}\left[-2(1+\omega)\left(r(-\omega)+r(-\lambda)\right)+2(\omega+\lambda)\right], && (41)\\
F_{0,cl}^{VP} &= -iK\left[-2\kappa\left(r(-\omega)+r(-\lambda)\right)+2(\kappa+1)\right], && (42)\\
F_{0,cl}^{PV} &= -iK\left[2\kappa\left(r(-\omega)+r(-\lambda)\right)+2(\kappa+1)\right], && (43)\\
F_{0,cl}^{VV} &= iK\sqrt{\frac{M_D^*}{M_D}}\left(2(\omega+1)\right), && (44)\\
F_{1,cl}^{VV} &= iK\sqrt{\frac{M_D^*}{M_D}}\Big((\kappa+1)2\left[r(-\omega)+r(-\lambda)\right]-2(\omega+\lambda)(H+G)-2\kappa^2\Big), && (45)\\
F_{2,cl}^{VV} &= -iK\sqrt{\frac{M_D^*}{M_D}}2\kappa, && (46)
\end{aligned}
$$

where

$$
K\equiv\frac{1}{2}C_2\,\zeta(\omega)\alpha_D\left(\frac{g_A}{f}\right)^2 I_L\sqrt{M_B M_{D^*} M_D}\,,\qquad I_L\equiv\frac{-im_K^2}{16\pi^2}\ln\frac{m_K^2}{\mu^2}\,, \qquad (47)
$$

$$
G\equiv\frac{(r(-\omega)+\omega)}{1-\omega^2}\kappa^2-\frac{(1+\omega\,r(-\omega))}{1-\omega^2}\kappa\ \ ,\quad H\equiv-\frac{(1+\lambda\,r(-\lambda))}{1-\lambda^2}\,. \qquad (48)
$$

Note that K is a priori proportional to the Wilson coefficient combination (C_2+C_1/N_c), as the amplitude in (29). But because the factor $1/f^2$ is already of order $1/N_c$, we replace (C_2+C_1/N_c) by just C_2 in (47).

4 Results and discussion

In our calculation we used the following input parameters: $\alpha_B=\alpha_D=0.33$ GeV$^{-3/2}$, $\rho=1.05$, $G_H=7.5$ GeV$^{-1/2}$ and $\langle\frac{\alpha_s}{\pi}G^2\rangle=[(0.315\pm0.020)\ \text{GeV}]^4$ [12, 13], $g_{\mathcal{A}}=0.6$ [23], $f_\pi=0.093$ GeV [3] and $\kappa=0.37$.

i	$F^{PP}_{0,i} \times 10^7$	$F^{VP}_{0,i} \times 10^7$	$F^{PV}_{0,i} \times 10^7$	$F^{VV}_{0,i} \times 10^7$	$F^{VV}_{1,i} \times 10^7$	$F^{VV}_{2,i} \times 10^7$
fc	0	0.12i	-0.12i	0	0	-0.12i
gc	-0.17-0.71i	1.11 + 0.14i	-0.51 + 0.87i	0.13-0.22i	-0.13+0.06i	0.22 -0.09i
cl	0.91-1.20i	-0.23 + 0.19i	-0.08 - 0.21i	0.56+0.03i	-0.24-0.45i	-0.09

Table 1: Reduced amplitudes for the $B^0 \to M\bar{M}$, $M = D_s^+, D_s^{+*}$ decay modes. The results are given in GeV.

i	$F^{PP}_{0,i} \times 10^7$	$F^{VP}_{0,i} \times 10^7$	$F^{PV}_{0,i} \times 10^7$	$F^{VV}_{0,i} \times 10^7$	$F^{VV}_{1,i} \times 10^7$	$F^{VV}_{2,i} \times 10^7$
fc	0	0.51i	-0.51i	0	0	-0.51i
gc	-0.74-3.16i	4.95 +0.63i	-2.27 +3.69i	0.60-1.00i	-0.59+0.25i	0.99 -0.42i
cl	3.78-4.96i	-0.96 +0.78i	-0.34 - 0.85i	2.13+0.10i	-0.91-1.72i	-0.34 -0.02i

Table 2: Reduced amplitudes for the $B_s^0 \to N\bar{N}$, $N = D^+, D^{+*}$ decay modes. The results are given in GeV.

The reduced amplitudes for $B^0 \to M\bar{M}$, $M = D_s^+, D_s^{+*}$ are presented in Table 1. We find the following branching ratios:

$$Br(\bar{B}^0 \to D_s^+ D_s^-) = 2.5 \times 10^{-4}\,, \qquad Br(\bar{B}_s^0 \to D^+ D^-) = 4.5 \times 10^{-3}\,, \tag{49}$$

$$Br(\bar{B}^0 \to D_s^{+*} D_s^-) = 3.3 \times 10^{-4}\,, \qquad Br(\bar{B}_s^0 \to D^{+*} D^-) = 6.8 \times 10^{-3}\,, \tag{50}$$

$$Br(\bar{B}^0 \to D_s^+ D_s^{-*}) = 2.0 \times 10^{-4}\,, \qquad Br(\bar{B}_s^0 \to D^+ D^{-*}) = 4.3 \times 10^{-3}\,, \tag{51}$$

$$Br(\bar{B}^0 \to D_s^{*+} D_s^{-*}) = 5.4 \times 10^{-4}\,, \qquad Br(\bar{B}_s^0 \to D^{*+} D^{-*}) = 9.1 \times 10^{-3}\,. \tag{52}$$

The contribution of the constant term and the corresponding counterterm can change the branchig ratio for B-meson decaying into two pseudoscalars by about 10%, while in the case of decay into one pseudoscalar and one vector D-meson, this contribution is in the range of $20 - 40\%$. In the case of B-meson decaying into two vector mesons, the constant term is estimated to be 2-8 times larger than the logaritmic contribution, depending on the choice of the scheme in which the products of two Levi-Civita terms are considered. The uncertainty in input parameters can result in an additional error for the branching ratios. We estimate that it can be of the order of 20%. Within our approach the $1/m_Q$ corrections, with $Q = c, b$ have been neglected. At least the $1/m_c$ corrections might be important.

The study of dominant contributions in the $\bar{B}_d^0 \to D_s^+ D_s^-$, $\bar{B}_d^0 \to D_s^{+*} D_s^-$ and $\bar{B}_d^0 \to D_s^+ D_s^{-*}$ decay amplitudes is very important for our understaning of the color suppressed contributions to the B-mesons decaying to two charm mesons. This is even more important knowing that the experimental rates for B_d decay into $D^- D_s^+$, $D^{-*} D_s^+$, $D^- D_s^{*+}$ and $D^{*-} D_s^{*+}$ have very small color suppressed contributions and therefore the decay amplitudes we consider open a window for studies of color suppressed effects in B-decays to two charm mesons. The chance for experimental measurements of these decay rates at B-factories makes the study of the decay mechanisms in these decays even more important.

5 Appendix A: Chiral loops with "superpropagator"

In the following, we will shortly present a "superpropagator method" which enables us to calculate all contributions of one kind of topology to all decays of a type $\bar{B}^0 \to M\bar{M}$, $M = D_s^*, D_s^{+*}$ as one compact calculation. This means that instead of calculating all the diagrams in Fig. 3, we only need to calculate two contributions, each one coming from one topology.

The superpropagator is a propagator of the $H^{(\pm)}(x)$ field and therefore includes propagators of both heavy pseudo-scalar and heavy vector meson. It is defined in a standard way as a contraction of $H^{(\pm)}(x)_{\alpha\beta}$ and $\overline{H^{(\pm)}}(y)_{\kappa\lambda}$, where $\alpha, \beta, \kappa, \lambda$ are Dirac spinor indices. In momentum space, we obtain the superpropagator

$$S^{(\pm)}_{\alpha\beta;\kappa\lambda}(k) \;=\; \frac{1}{2(\pm v\cdot k + i\epsilon)}\, T^{(\pm)}_{\alpha\beta;\kappa\lambda}(v)\,, \tag{53}$$

where k is the momentum and

$$\begin{aligned} T^{(+)}_{\alpha\beta;\kappa\lambda}(v) \;&=\; (P_+(v)\gamma_\tau)_{\alpha\beta}\,(-g^{\tau\nu} + v^\tau v^\nu)\,(\gamma_\nu P_+(v))_{\kappa\lambda} \\ &\quad - (P_+(v)\gamma_5)_{\alpha\beta}\;(\gamma_5 P_+(v))_{\kappa\lambda}\;, \end{aligned} \tag{54}$$

$$\begin{aligned} T^{(-)}_{\alpha\beta;\kappa\lambda}(v) \;&=\; (\gamma_\tau P_-(v))_{\alpha\beta}\,(-g^{\tau\nu} + v^\tau v^\nu)\,(P_-(v)\gamma_\nu)_{\kappa\lambda} \\ &\quad - (\gamma_5 P_-(v))_{\alpha\beta}\;(P_-(v)\gamma_5)_{\kappa\lambda}\;. \end{aligned} \tag{55}$$

For the effective Lagrangian of topology I we have:

$$\begin{aligned} \mathcal{L}_I(B\to D\bar{D}) \;&=\; -i\,C_2\,\zeta(\omega)\frac{\alpha_D}{2}(\frac{g_A}{f})^2\; I(v_b,-\overline{v})^{\sigma\rho}\left[\overline{H_c^{(+)}}\gamma^\mu L\right]_{\beta\alpha} T^{(+)}_{\alpha\beta;\kappa\lambda}(v_b) \\ &\quad\times\;\left[H_b^{(+)}\gamma_\sigma\gamma_5\right]_{\lambda\kappa}\left[H_{\bar{c}}^{(-)}\gamma_\rho\gamma_5\right]_{\eta\delta} T^{(-)}_{\delta\eta;\phi\xi}(\overline{v})\;\;[\gamma_\mu L]_{\xi\phi}\quad, \end{aligned} \tag{56}$$

while the effective Lagrangian of topology II can be written as:

$$\begin{aligned} \mathcal{L}_{II}(B\to D\bar{D}) \;&=\; -i\,C_2\,\zeta(\omega)\frac{\alpha_D}{2}(\frac{g_A}{f})^2\; I(v_c,-\overline{v})^{\sigma\rho}\left[\gamma_\sigma\gamma_5\overline{H_c^{(+)}}\right]_{\beta\alpha} T^{(+)}_{\alpha\beta;\kappa\lambda}(v_c) \\ &\quad\times\;\left[\gamma_\mu L H_b^{(+)}\right]_{\lambda\kappa}\left[H_{\bar{c}}^{(-)}\gamma_\rho\gamma_5\right]_{\eta\delta} T^{(-)}_{\delta\eta;\phi\xi}(\overline{v})\;\;[\gamma_\mu L]_{\xi\phi}\;. \end{aligned} \tag{57}$$

From (56) and (57) one can derive the (reduced) amplitudes already given in (41)-(46).

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